Spherical geometry

13

syllabus reference

Measurement 7
- **Spherical geometry**

In this chapter

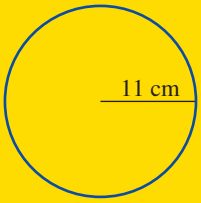
Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest HSC Course* CD-ROM or ask your teacher for a copy.

13.1 Circumference of a circle

1 Find the circumference of the following circles. Answer correct to 1 decimal place.

a



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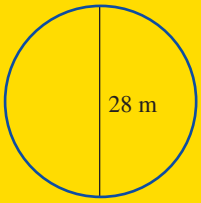

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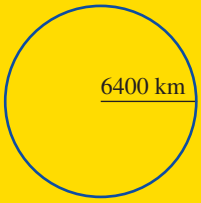
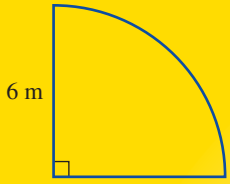


13.2 Calculating arc length

2 Find the arc length of each of the following. Answer correct to 3 significant figures.

a



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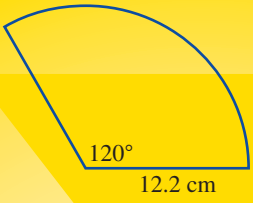

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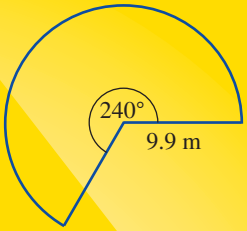


13.3 Converting units of time

3 Convert each of the following to the unit indicated in brackets.

a 160 minutes (hours and minutes)
b $3\frac{1}{2}$ hours (minutes)
c 4 weeks (days)
d 15 months (years)

4 Use 1 nautical mile $\simeq$ 1.852 km to convert the following.

a 6 nautical miles to kilometres
b 80 kilometres to nautical miles
c 4.2 nautical miles to metres
d 7530 metres to nautical miles

Arc lengths

An arc is a section of the circumference of a circle. To calculate an arc length we must first revise the circumference of a circle. The circumference of a circle can be found using either of the formulas:

$C = \pi d$, where d is the diameter
$C = 2\pi r$, where r is the radius.

WORKED Example 1

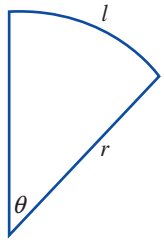
Calculate the circumference of a circle that has a radius of 6 m. Give your answer correct to 2 decimal places.

THINK	WRITE
1 Write the formula.	$C = 2\pi r$
2 Substitute the value of r.	$C = 2 \times \pi \times 6$
3 Calculate the circumference.	$C = 37.70$ m

The length of an arc can be calculated as the fraction of the circle determined by the angle subtended by the arc at the centre, as shown in the figure on the right.

The arc length, l, can be calculated using the formula:

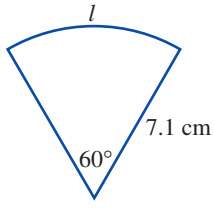
$$l = \frac{\theta}{360} 2\pi r$$



where θ = number of degrees in the central angle.

WORKED Example 2

Calculate the length of the arc shown on the right, correct to 1 decimal place.



THINK	WRITE
1 Write the formula.	$l = \frac{\theta}{360} 2\pi r$
2 Substitute the value of θ and r.	$= \frac{60}{360} \times 2 \times \pi \times 7.1$
3 Calculate the arc length.	$= 7.4$ cm

The arc length formula is then used to make calculations about the distance between points on the Earth's surface.

WORKED Example 3

The radius of the Earth at the equator is approximately 6400 km.

a Calculate the circumference of the Earth at the equator, correct to the nearest kilometre.

b Two points on the equator subtend at a 5° angle at the centre of the Earth. Calculate the distance between them, correct to the nearest kilometre.

THINK	WRITE
a 1 Write the formula.	**a** $C = 2\pi r$
2 Substitute the value of r.	$= 2 \times \pi \times 6400$
3 Calculate the circumference.	$= 40\,212$ km
b 1 Write the formula.	**b** $d = \frac{\theta}{360} 2\pi r$
2 Substitute for θ and r.	$= \frac{5}{360} \times 2 \times \pi \times 6400$
3 Calculate the distance.	$= 559$ km

remember

1. The circumference of a circle can be found using either of the formulas $C = \pi d$ or $C = 2\pi r$.
2. An arc length is calculated by using the angle the arc subtends at the centre of a circle. The arc length is calculated using the formula:
$$l = \frac{\theta}{360} 2\pi r$$ FS
where θ is the number of degrees in the central angle.
3. The arc length formula can be used to find the distance between points on the equator.

EXERCISE 13A Arc lengths

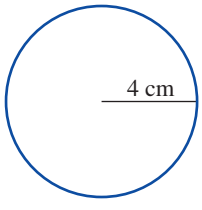
SkillSHEET 13.1 Circumference of a circle

WORKED Example 1

Cabri Geometry Arc length

1 Calculate the circumference of each of the following circles, correct to 1 decimal place.

a



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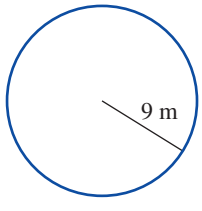

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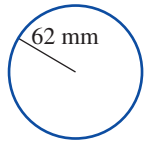
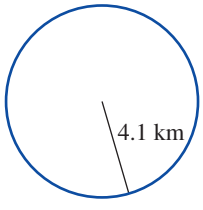


d



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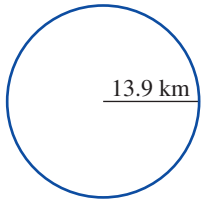

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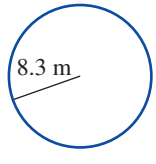
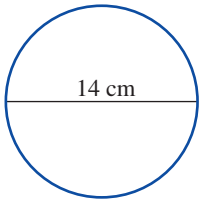


2 Calculate the circumference of a circle with a radius of 100 km. Give your answer correct to the nearest 10 km.

3 Calculate the circumference of each of the following circles. Give your answer correct to 3 significant figures.

a



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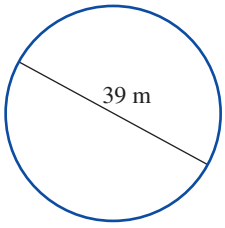

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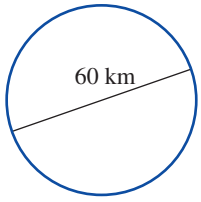

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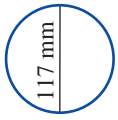

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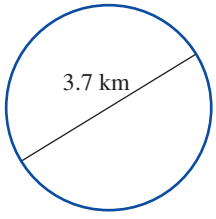

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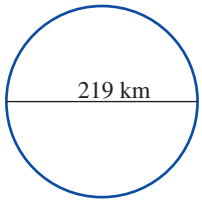


4 Calculate the circumference of a circle that has a diameter of 3000 km. Give your answer correct to the nearest 100 km.

5 Use the formula $C = \pi d$ to find the diameter of a circle with a circumference of 100 cm. Give your answer correct to 2 decimal places.

6 Find (correct to 3 significant figures):

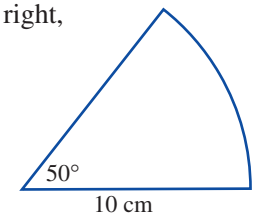
a the diameter of a circle with a circumference of 80 m

b the radius of a circle with a circumference of 42.3 cm

c the diameter of a sphere with a circumference of 2500 km.

WORKED Example 2

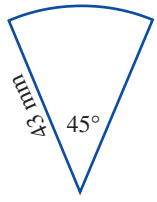
7 Calculate the length of the arc shown on the right, correct to 2 decimal places.



8 Calculate the lengths of each of the arcs drawn below, correct to 1 decimal place.

13.2 SkillSHEET Calculating arc length

a



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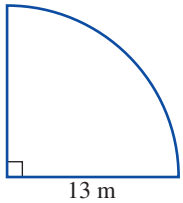

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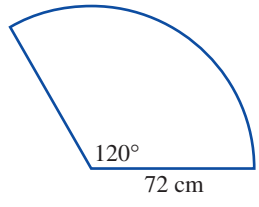

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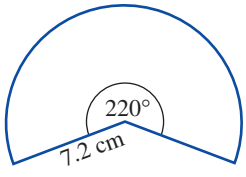

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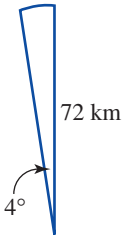

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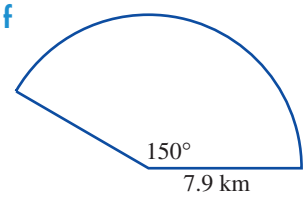


WORKED Example 3

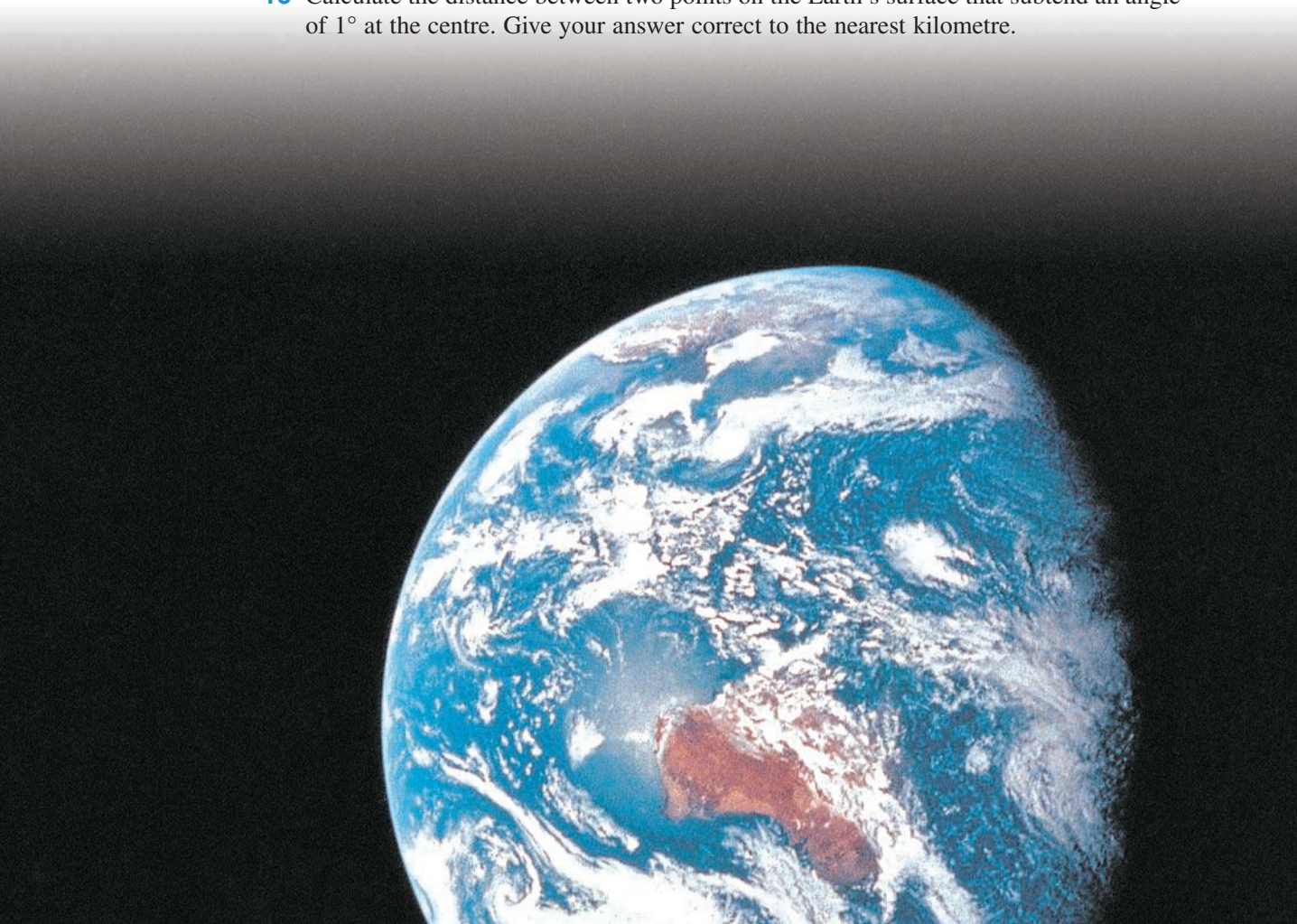
9 The radius of the Earth at the equator is 6400 km.
- **a** Calculate the circumference of the Earth at the equator, correct to the nearest 100 km.
- **b** Two points on the equator subtend an angle of 40° at the centre. Calculate the distance between them, correct to the nearest 10 km.

10 A circle has a radius of 30 km.
- **a** Two points on the circle subtend an angle of 100° at the centre. Calculate the length of the arc joining them, correct to the nearest kilometre.
- **b** The outer arc between the two points subtends an angle of 260°. Calculate the length of the outer arc, correct to the nearest kilometre.
- **c** Show that the sum of the lengths of the two arcs is equal to the circumference of the circle.

11 A sphere has a radius of 40 cm.
- **a** Calculate the circumference of the sphere, correct to the nearest centimetre.
- **b** Calculate the distance between two points on the sphere that subtend an angle of 90° at the centre, correct to 1 decimal place.

12 Calculate the distance, correct to 1 decimal place, between two points on a sphere:
- **a** of radius 10 cm, which subtend an angle of 30° at the centre
- **b** of radius 2 m, which subtend an angle at 122° at the centre
- **c** of radius 6400 km, which subtend an angle of 51° at the centre.

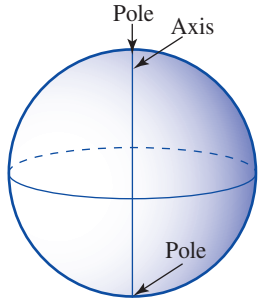
13 Calculate the distance between two points on the Earth's surface that subtend an angle of 1° at the centre. Give your answer correct to the nearest kilometre.

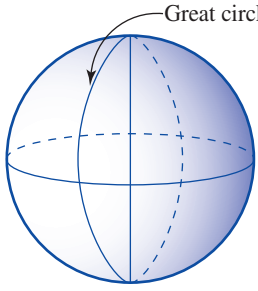


Great circles and small circles

Consider the sphere drawn on the right. The axis of the sphere is a diameter of that sphere. The ends of the axis are called the *poles*.

If we draw any lines around the sphere passing through both poles, a **great circle** is formed. A great circle is the largest possible circle that can be drawn around the sphere.





The length of a great circle is found using the formulas for the circumference of a circle:

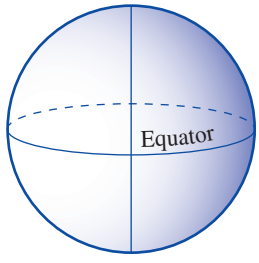
$C = \pi d$, where d is the diameter of the sphere
or $C = 2\pi r$, where r is the radius of the sphere.

WORKED Example 4

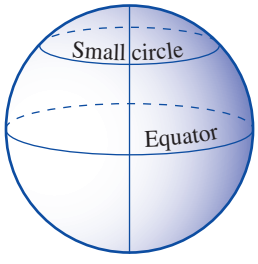
Calculate the length of a great circle on a sphere with a radius of 40 cm. Give your answer correct to the nearest centimetre.

THINK	WRITE
1 Write the formula.	$C = 2\pi r$
2 Substitute the radius of the sphere.	$= 2 \times \pi \times 40$
3 Calculate the length of the great circle.	$= 251$ cm

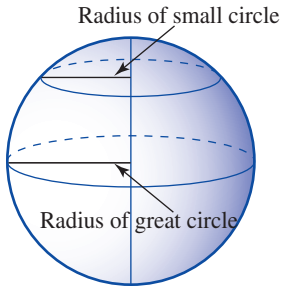
Now consider a circle drawn perpendicular to the axis of the sphere. Only one circle, called the equator, will be a great circle. The centre of the equator will be the centre of the sphere as shown on the right.



Other circles that are perpendicular to the axis of the sphere will be smaller than a great circle and are called **small circles**.

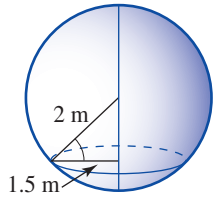


To calculate the length around a small circle, we need to know the small circle's radius. The small circle will have a radius smaller than that of the great circle, as shown in the figure on the right.



WORKED Example 5

Calculate the length of the small circle on the right, correct to 1 decimal place.



THINK	WRITE
1 Write the formula for the circumference of a circle.	$C = 2\pi r$
2 Substitute for r.	$= 2 \times \pi \times 1.5$
3 Calculate the circumference.	$= 9.4$ cm

remember

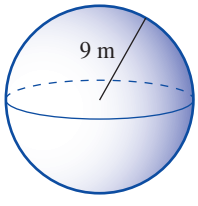
1. A great circle is the circle of the greatest possible size that lies on the surface of a sphere.
2. The length of a great circle is calculated using the formula for the circumference of a circle.
3. A small circle is any circle smaller in size than the great circle.

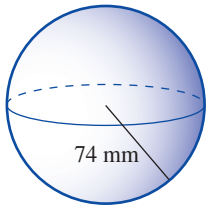
EXERCISE 13B Great circles and small circles

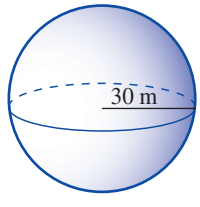
WORKED Example 4

1 Calculate the length of a great circle around a sphere of radius 7 cm. Give your answer correct to 2 decimal places.

2 Calculate the length of a great circle on each of the spheres drawn below. Give each answer correct to 3 significant figures.

a 

b 

c 

3 The Earth is a sphere with a radius of approximately 6400 km. Calculate the length of a great circle around the Earth's surface, correct to the nearest 10 km.

4 Calculate the length of a great circle that lies on a sphere with a diameter of 1 m. Give your answer correct to the nearest centimetre.

5 Below are the diameters of the other planets in our solar system. Calculate the length of a great circle on the surface of each planet, correct to the nearest 10 km.

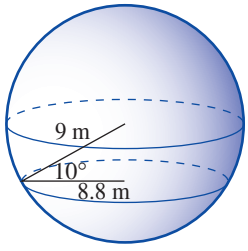
a Mercury 4878 km **b** Venus 12 100 km **c** Mars 6796 km
d Jupiter 142 984 km **e** Saturn 120 540 km **f** Uranus 51 118 km
g Neptune 49 100 km

WORKED Example 5

6 Calculate the length of a small circle on a sphere of radius 4 m if the radius of the small circle is 2 m. Give your answer correct to 2 decimal places.

7 Calculate the length of each of the small circles below, correct to 1 decimal place.

a



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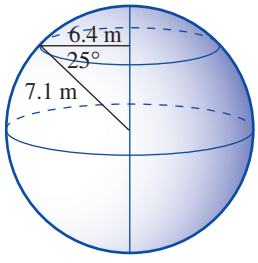

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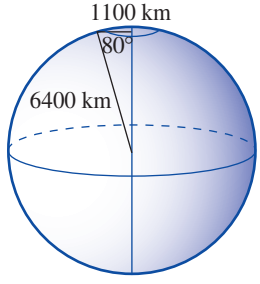
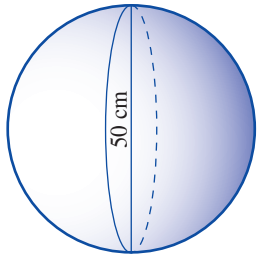


8 **multiple choice**

The diameter of the moon is 3476 km. The length of a great circle on the moon's surface is closest to:

A 5460 km **B** 10 920 km **C** 21 840 km **D** 43 680 km

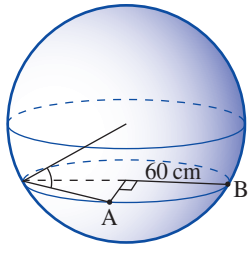
9 The diameter of the sphere drawn on the right is 50 cm. Calculate the distance along the surface from one pole to the other. Give your answer correct to the nearest centimetre.



10 Calculate the distance between the north and south poles along the Earth's surface, correct to the nearest 100 km. (Take the radius of the Earth to be 6400 km.)

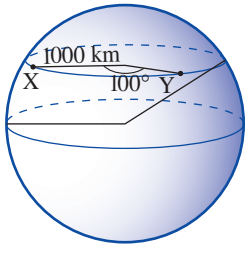
11 A and B are two points on a small circle of radius 60 cm, as shown in the figure on the right.

a Calculate the circumference of the small circle, correct to the nearest centimetre.
b Calculate the length of the arc, AB, correct to the nearest centimetre.

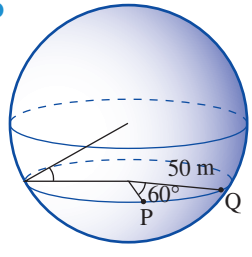


12 Calculate the length of each arc below, correct to 3 significant figures.

a



b



WORKSHEET 13.1

10 QUICK QUESTIONS 1

1 Calculate the circumference of a circle that has a radius of 4.7 m. Give your answer correct to 1 decimal place.

2 Calculate the circumference of a circle with a diameter of 56 cm. Give your answer correct to 2 significant figures.

3 An arc on a circle of radius 9 cm subtends an angle of 60° at the centre. Calculate the length of the arc, correct to 1 decimal place.

4 What is a great circle?

5 What is a small circle?

6 Calculate the length of a great circle that lies on a sphere of radius 15 cm. Give your answer correct to 2 decimal places.

7 Calculate the distance between the north and south poles on a planet that has a diameter of 4500 km. Give your answer correct to the nearest 100 km.

8 Calculate the distance between two points on the great circle of the planet in question 7 that subtend an angle of 80° at the centre. Give your answer correct to the nearest 10 km.

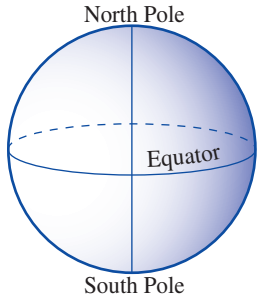
9 A planet has a radius of 2300 km. Calculate the length of a great circle lying on the surface of the planet, correct to the nearest 10 km.

10 Sydney lies on a small circle of the Earth that has a radius of approximately 5400 km. Calculate the circumference of the small circle (correct to the nearest 100 km).

Latitude and longitude

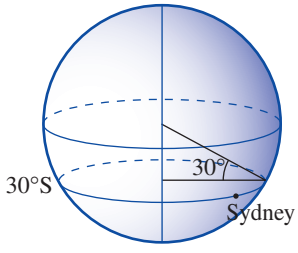
As the Earth is a sphere, great circles and small circles on the surface of the Earth are used to locate points on the surface.

Consider the axis of the Earth to be the diameter joining the North Pole and the South Pole. The only great circle that is perpendicular to this axis is the equator. The angular distance either north or south of the equator is the **latitude**.



Small circles parallel to the equator are called **parallels of latitude**. These small circles are used to describe how far north or south of the equator a place is. For example Sydney lies close to the small circle 30°S.

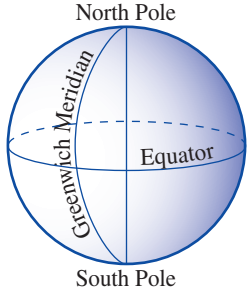
This means Sydney subtends a 30° angle at the centre of the Earth and is south of the equator.



The maximum latitude for any point on the Earth is 90° N or 90° S. The north and south poles lie at these points.

For latitude, the equator is the line of reference for all measurements.

To locate a place on the globe in an east–west direction, the line of reference is the **Greenwich Meridian**. The Greenwich Meridian is half a great circle running from the North to the South Pole.

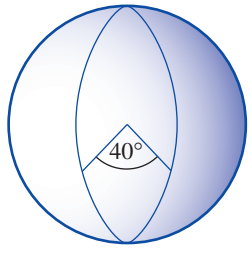


The Greenwich Meridian is named after Greenwich, a suburb of London through which the circle runs.

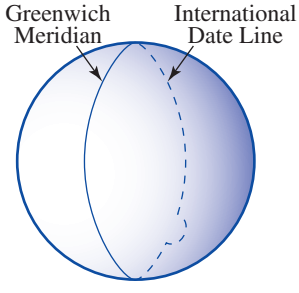
All other places on the globe are located by the half great circle on which they lie. These half great circles are called **meridians of longitude**.

Each meridian of longitude is identified by the angle between it and the Greenwich Meridian and by whether it is east or west of Greenwich.

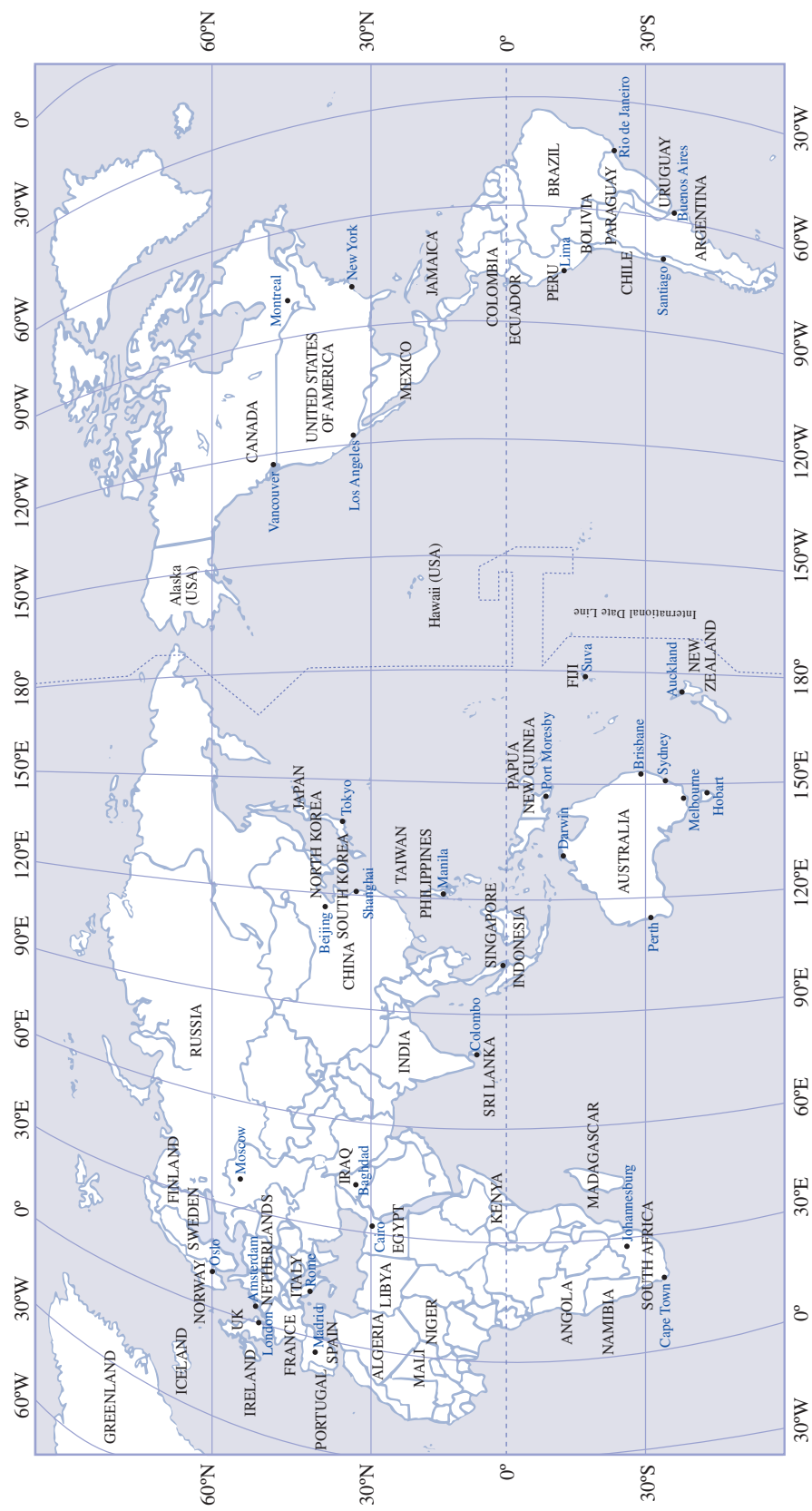
The meridian of longitude opposite the Greenwich Meridian is the **International Date Line**. The International Date Line has longitude 180° either east or west. On either side of the International Date Line the day changes. (This will be explained in more detail later in the chapter.)



For the convenience of some small island nations and Russia, the International Date Line is bent so not to pass through them.



World maps or globes are drawn with both parallels of latitude and meridians of longitude shown. Any location on a map or globe can be giv en a pair of coordinates: the first is the parallel of latitude that it lies on, the second is the meridian of longitude. For example, the coordinates of Sydney are 30°S, 150°E.


60°N
30°N
0°
30°S
60°W
30°W
0°
30°E
60°E
90°E
120°E
150°E
180°
150°W
120°W
90°W
60°W
30°W
0°
GREENLAND
ICELAND
IRELAND
UK
London
NORWAY
SWEDEN
Oslo
FINLAND
Moscow
RUSSIA
NETHERLANDS
Amsterdam
FRANCE
ITALY
Rome
Madrid
SPAIN
PORTUGAL
ALGERIA
LIBYA
Cairo
EGYPT
MALI
NIGER
IRAQ
Baghdad
INDIA
Colombo
SRI LANKA
CHINA
Beijing
NORTH KOREA
SOUTH KOREA
Shanghai
Tokyo
JAPAN
TAIWAN
PHILIPPINES
Manila
SINGAPORE
INDONESIA
PAPUA NEW GUINEA
Port Moresby
Darwin
AUSTRALIA
Perth
Brisbane
Sydney
Melbourne
Hobart
FIJI
Suva
NEW ZEALAND
Auckland
International Date Line
Hawaii (USA)
Alaska (USA)
CANADA
Vancouver
Montreal
UNITED STATES OF AMERICA
Los Angeles
New York
MEXICO
JAMAICA
COLOMBIA
ECUADOR
PERU
Lima
BRAZIL
Rio de Janeiro
BOLIVIA
PARAGUAY
CHILE
Santiago
URUGUAY
Buenos Aires
ARGENTINA
KENYA
MADAGASCAR
Johannesburg
ANGOLA
NAMIBIA
SOUTH AFRICA
Cape Town

WORKED Example 6

Identify the major cities closest to each of the following locations using the map on page 394.

a 30°S, 30°E **b** 30°N, 120°E **c** 45°N, 75°W

THINK	WRITE
a Look for the city closest to the intersection of the 30°S parallel of latitude and the 30°E meridian of longitude.	**a** Johannesburg
b Look for the city closest to the intersection of the 30°N parallel of latitude and the 120°E meridian of longitude.	**b** Shanghai
c Look for the city closest to the intersection of the 45°N parallel of latitude and the 75°W meridian of longitude.	**c** Montreal

WORKED Example 7

Write down the approximate coordinates of each of the following cities using the map on page 394.

a Singapore **b** Perth **c** Los Angeles

THINK	WRITE
a 1 Use the parallels of latitude drawn to estimate the latitude. 2 Use the meridians of longitude drawn to estimate the longitude.	**a** 1°N, 104°E
b 1 Use the parallels of latitude drawn to estimate the latitude. 2 Use the meridians of longitude drawn to estimate the longitude.	**b** 32°S, 115°E
c 1 Use the parallels of latitude drawn to estimate the latitude. 2 Use the meridians of longitude drawn to estimate the longitude.	**c** 35°N, 118°W

remember

1. The Earth's axis runs from the North Pole to the South Pole.
2. The only great circle perpendicular to the Earth's axis is the equator.
3. Places are located either north or south of the equator using parallels of latitude. Each parallel of latitude is a small circle parallel to the equator and is identified by the angle it subtends at the centre.
4. Places are located as being east or west of the Greenwich Meridian. The Greenwich Meridian is half a great circle from the North Pole to the South Pole.
5. All meridians of longitude are half great circles and are measured by the angle made to the east or west of the Greenwich Meridian.
6. A pair of coordinates can identify every point on the Earth's surface. These are called its latitude and longitude.

EXERCISE 13C Latitude and longitude

For the following questions use the map on page 394.

WORKED Example 6

1 Write down the name of the city closest to each of the following pairs of coordinates.

a 30°N, 30°E
b 30°N, 120°E
c 15°S, 135°E
d 45°N, 75°W
e 50°N, 0°
f 37°S, 175°E
g 35°N, 140°E
h 40°N, 115°E
i $22\frac{1}{2}$°S, 43°W
j 60°N, 11°E

WORKED Example 7

2 State the approximate latitude and longitude of each of the following major cities or islands.

a Melbourne
b New York
c Jamaica
d Johannesburg
e Rome
f Buenos Aires
g Baghdad
h Moscow
i Singapore
j Suva

Important parallels of latitude

Four significant parallels of latitude on the surface of the Earth are the:

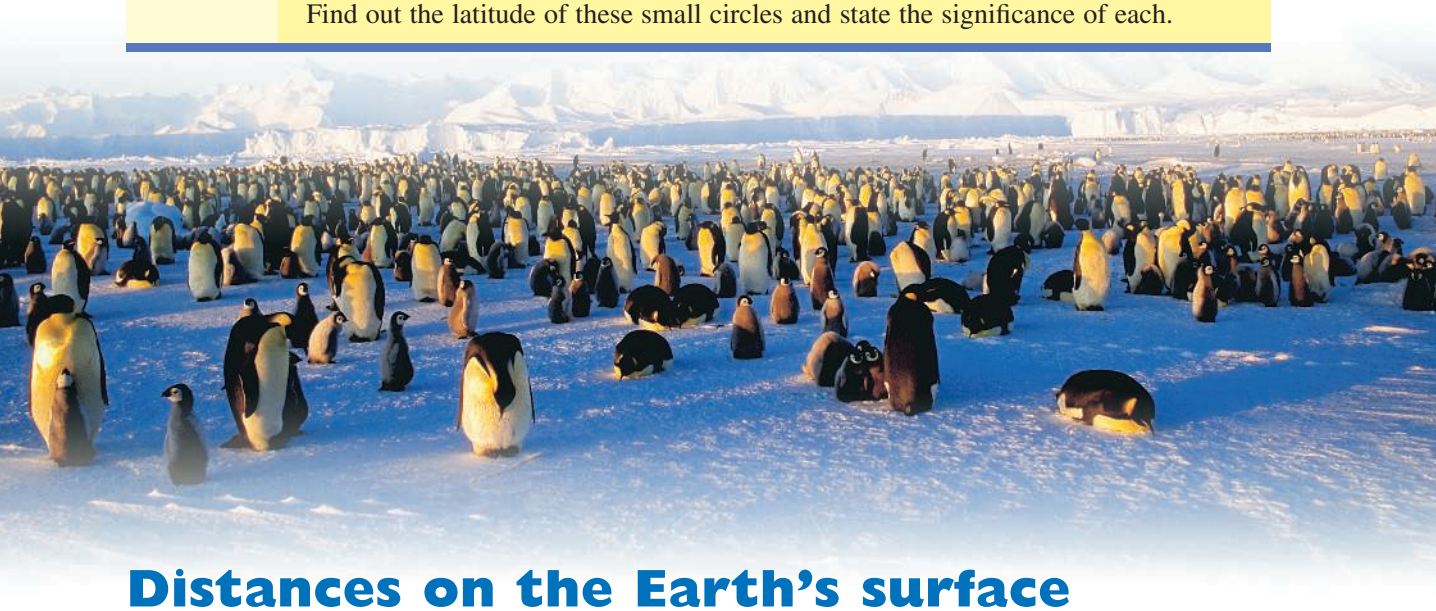
1 Arctic Circle

2 Antarctic Circle

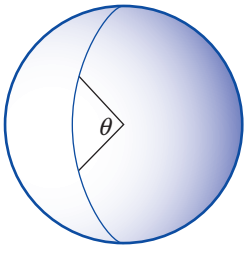
3 Tropic of Cancer

4 Tropic of Capricorn.

Find out the latitude of these small circles and state the significance of each.



Distances on the Earth's surface



From the previous section on latitude and longitude it can be seen that angular geometry is of great importance when making measurements on the Earth's surface.

Now consider a meridian of longitude on the Earth's surface with two points on it. The angular distance between them will be the difference between their latitudes.

The angular distance is calculated by subtracting the latitudes of points if both are on the same side of the equator and adding the latitudes if on opposite sides of the equator.

WORKED Example 8

The coordinates of A are (20°S, 130°E) and the coordinates of B are (15°N, 130°E). Find the angular distance between them.

THINK

A and B are on opposite sides of the equator so add the latitudes.

WRITE

Angular distance = 20° + 15°
= 35°

Now consider two points on the same great circle that have an angular distance of 1 minute. (Remember $60' = 1°$.) The distance between these two points is defined to be 1 **nautical mile (M)**. Therefore, an angular distance of 1^o on a great circle will equal 60 nautical miles.

1 nautical mile ≃ 1.852 km

Using this information, we are able to calculate the distance between two points on a great circle on the Earth's surface in both nautical miles and kilometres.

WORKED Example 9

P and Q are two points on the Earth's surface with coordinates (27°N, 30°W) and (39°N, 30°W) respectively.
a Calculate the distance between P and Q in nautical miles.
b Use 1 M ≃ 1.852 km to give the distance, PQ, to the nearest kilometre.

THINK	WRITE
a 1 P and Q are on the same great circle.	**a**
2 Calculate the angular distance, PQ.	Angular distance = 39° − 27°
	= 12°
3 Convert the angular distance to nautical miles using 1° = 60 M.	Distance = 12 × 60
	= 720 M
b Multiply 720 by 1.852 to convert to kilometres.	**b** Distance = 720 × 1.852
	= 1333 km

We can also calculate the distance between two points on the same great circle, in kilometres, using the fact that the radius of the Earth is 6400 km.

WORKED Example 10

X and Y are two points on the Earth's surface with coordinates (32°N, 120°E) and (45°S, 120°E). Calculate the distance, XY, correct to the nearest 100 km.

THINK	WRITE
1 Calculate the angular distance, XY.	Angular distance = 32° + 45°
	= 77°
2 Use the arc length formula to calculate the distance between X and Y, correct to the nearest 100 km.	$l = \frac{\theta}{360} 2\pi r$
	$= \frac{77}{360} \times 2\pi r$
	$= \frac{77}{360} \times 2 \times \pi \times 6400$
	= 8600 km

In marine and aerial navigation, speed on the Earth's surface is measured in knots.

1 knot = 1 nautical mile/hour

WORKED Example 11

The coordinates of two points on the Earth's surface are given by the coordinates A(50°N, 120°W) and B(30°S, 120°W). Calculate the time taken for a ship to sail the shortest distance between these two points at an average speed of 40 knots.

THINK

1. Calculate the angular distance between A and B.
2. Calculate the distance, AB, in nautical miles.
3. Use the formula time $= \frac{\text{distance}}{\text{speed}}$ to calculate the time taken for the journey.

WRITE

Angular distance $= 50° + 30°$
$= 80°$

Distance $= 80 \times 60$
$= 4800$ M

Time $= \frac{\text{distance}}{\text{speed}}$
$= \frac{4800}{40}$
$= 120$ hours (5 days)

remember

1. The angular distance between two points on the same great circle (meridian of longitude) can be found by:
   - subtracting the latitudes if the points are on the same side of the equator
   - adding the latitudes if the points are on opposite sides of the equator.
2. An angular distance can be converted to a linear distance using the relationship, 1° = 1 nautical mile (M).
3. 1 M $\simeq$ 1.852 km
4. The distance between two points on the same great circle can also be found using the arc length formula and taking the radius of the Earth as being 6400 km.
5. Speed can be measured in knots. A speed of 1 knot = 1 M/hour.

EXERCISE 13D Distances on the Earth's surface

WORKED Example 8

1 Two points, A and B, on the Earth's surface are at (30°N, 25°W) and (20°S, 25°W). Calculate the angular distance between A and B.

2 In each of the following calculate the angular distance between the pairs of points given.

a (70°N, 150°E) and (30°N, 150°E)
b (25°N, 40°W) and (15°S, 40°W)
c (64°N, 0°) and (7°S, 0°)
d (42°S, 97°W) and (21°S, 97°W)
e (0°, 60°E) and (0°, 20°W)

3 The city of Durban is at approximately (30°S, 30°E) while Cairo is at approximately (30°N, 30°E). What is the angular distance between Durban and Cairo?

WORKED Example 9

4 P and Q are two points on the Earth's surface with coordinates (45°N, 10°W) and (15°N, 10°W) respectively.
- **a** Calculate the distance between P and Q in nautical miles (M).
- **b** Use 1 M = 1.852 km to calculate the distance, PQ, correct to the nearest km.

5 Calculate the distance between each of the points below in nautical miles.
- **a** A (10°N, 45°E) and B (25°S, 45°E)
- **b** C (75°N, 86°W) and D (60°S, 86°W)
- **c** E (46°S, 52°W) and F (7°S, 52°W)
- **d** G (34°N, 172°E) and H (62°S, 172°E)

6 The city of Osaka is at (37°N, 135°E) while Alice Springs is at (23°S, 135°E).
- **a** Calculate the distance between Osaka and Alice Springs in nautical miles.
- **b** Use 1 M = 1.852 km to write this distance, correct to the nearest kilometre.

7 The Tropic of Cancer is at latitude $22\frac{1}{2}$°N while the Tropic of Capricorn is at latitude $22\frac{1}{2}$°S. Calculate the distance between these two tropics along the same great circle in:
- **a** nautical miles
- **b** kilometres (correct to the nearest km).

WORKED Example 10

8 M and N are two points on the Earth's surface with coordinates (56°N, 122°W) and (3°S, 122°W). Calculate the distance, MN, correct to the nearest 100 km, using the arc length formula. (Take the radius of the Earth to be 6400 km.)

9 Calculate the distance between each of the points below, correct to the nearest kilometre, using the arc length formula and taking the radius of the Earth to be 6400 km.
- **a** P (85°S, 89°E) and Q (46°S, 89°E)
- **b** R (24°N, 0°) and S (12°S, 0°)
- **c** T (34°S, 17°W) and U (0°, 17°W)

10 **multiple choice**

Perth is at approximately (31°S, 115°E) while Hong Kong is at approximately (22°N, 115°E). The distance between Perth and Hong Kong is approximately:

A 9 M **B** 53 M **C** 540 M **D** 3180 M

11 **multiple choice**

Rachel is a flight navigator. She is responsible for calculating the distance between Stockholm (60°N, 18°E) and Budapest (47°N, 18°E). Rachel calculates the distance using the arc length formula, assuming the radius of the Earth is 6400 km. Rachel's answer would be closest to:

A 1445 km **B** 1452 km **C** 11 952 km **D** 11 890 km

WORKED Example 11

12 The coordinates of two points on the Earth's surface are X (40°S, 30°E) and Y (10°S, 30°E). Calculate the time taken for a plane to fly from X to Y at a speed of 240 knots.

13 Quito (0°, 78°W) and Kampala (0°, 32°E) are two cities on the Equator.
- **a** Calculate the angular distance between Quito and Kampala.
- **b** Calculate the distance between them in nautical miles.
- **c** Use 1 M = 1.852 km to find the distance, correct to the nearest 100 km.
- **d** Calculate the time taken to fly from Quito to Kampala at a speed of 480 knots.

14 Calculate the distance between the North Pole and the South Pole in nautical miles.

15 The city of Kingston is at approximately (18°N, 76°W). Ottawa is at approximately (46°N, 76°W).
- **a** Calculate the angular distance between Kingston and Ottawa.
- **b** Calculate the distance between Kingston and Ottawa in nautical miles.
- **c** Use 1 M = 1.852 km to calculate the distance, correct to the nearest kilometre.
- **d** Use the arc length formula to calculate the distance between the two cities, correct to the nearest kilometre.
- **e** Explain the discrepancy between the two answers.

10 QUICK QUESTIONS 2

1 Calculate the length of a great circle lying on the surface of a sphere of radius 24 cm. Give your answer correct to the nearest centimetre.

Using the map on page 394, give the coordinates of the following cities.

2 Amsterdam

3 Brisbane

4 Vancouver

Using the same map, state the name of the major city closest to the following coordinates.

5 (40°N, 10°E)

6 (12°N, 77°E)

Two points on the Earth's surface are P (13°N, 75°W) and Q (49°N, 75°W).

7 What is the angular distance between P and Q?

8 Calculate the distance, PQ, in nautical miles.

9 Use 1 M = 1.852 km to give the distance, PQ, correct to the nearest 10 km.

10 Use the arc length formula to calculate the distance, PQ, correct to the nearest 10 km.

Time zones

As the Earth rotates, different parts of the globe are experiencing day and night. This means that each meridian of longitude on the Earth's surface should have a different time of day. To simplify this, the Earth is divided into time zones.

Time zones are all calculated in relation to Greenwich. The time on the Greenwich Meridian is known as **Greenwich Mean Time (GMT)**. Time zones are then stated in terms of the number of hours they are ahead or behind GMT. All places with longitudes east of Greenwich are ahead of GMT, while all place with longitudes west of Greenwich are behind GMT. For example, Eastern Standard Time is GMT +10, meaning that Sydney is 10 hours ahead of GMT. When GMT is noon, EST is 10.00 pm.

The International Date Line is 12 hours ahead of Greenwich when travelling east and 12 hours behind when travelling west, so this totals 24 hours, or one day. Therefore, the day changes on either side of the International Date Line.

The time difference between two places is calculated by subtracting the comparative time with GMT.

WORKED Example 12

Sydney is GMT +10 while New York is GMT −5. Calculate the time difference between Sydney and New York.

THINK

1. Subtract the comparative times.
2. State the difference and which city is ahead in time.

WRITE

Time difference = 10 − (−5)
= 15
Sydney is 15 hours ahead of New York.

Once we have calculated the time difference, we are able to calculate the time in one place given the time in another. To calculate the time in a city further ahead of GMT we add time, or to calculate the time in a city further behind GMT we subtract time.

WORKED Example 13

Perth is GMT +8 while Cape Town is GMT +1. When it is 11.00 am in Cape Town, what is the time in Perth?

THINK

1. Calculate the time difference and state which city is ahead.
2. Add the time difference to the time in Cape Town to calculate the time in Perth.

WRITE

Time difference = 8 − 1
= 7 hours
Perth is 7 hours ahead of Cape Town.
Time in Perth = 11.00 am + 7 hours
= 6.00 pm

The time as calculated by the longitude is called the standard time. Time zones are calculated to approximate all the standard times within a region.

Australian time zones

At times when daylight saving time does not apply, Australia is divided into three time zones, Eastern Standard Time (EST), Central Standard Time (CST) and Western Standard Time (WST).

1. Which states are in each of the three time zones?
2. What is the time difference between each of these zones?
3. In which states does daylight saving time apply in summer?
4. When daylight saving is in force in each of the states that have daylight saving, how many time zones is Australia divided into?

As there are 24 hours in a day and 360° of longitude (180°E and 180°W), we can calculate that:

1 hour = 15° of longitude
1° = 4 minutes

We are now able to compare the time in various cities given the longitude of each.

WORKED Example 14

Calculate the time in Los Angeles (34°N, 120°W) when it is 8.00 am on Wednesday in Sydney (33°S, 150°E).

THINK	WRITE
1 Calculate the difference in longitudes.	Longitude difference = 150° + 120° = 270°
2 Convert this angular distance into hours using 1° = 4 minutes.	Time difference = 270 × 4 = 1080 minutes = 18 hours
3 Subtract the time difference from the time in Sydney.	Time in Los Angeles = 8.00 am Wednesday − 18 hours = 2.00 pm Tuesday

It is important to note that, for convenience, places that have almost the same longitude have the same time. An example of this is Australia's time zones where all of Queensland, New South Wales, Victoria and Tasmania are in the same standard time zone although there is a difference of 12° in longitude from the easternmost and westernmost points in this zone.

These calculations can then be used to calculate the arrival and departure times for international travel.

WORKED Example 15

A plane leaves London (50°N, 0°) at 9.00 am Sunday, London time, and flies to Sydney (33°S, 150°E). The flight takes 20 hours. Calculate the time in Sydney when the plane arrives.

THINK	WRITE
1 Calculate the longitude difference between Sydney and London.	Longitude difference = 150° − 0° = 150°
2 Use 1° = 4 minutes to calculate the time difference.	Time difference = 150 × 4 = 600 minutes = 10 hours
3 Calculate the time in Sydney when the plane is departing London by adding the time difference.	When the plane leaves London at 9.00 am (London time) Time in Sydney = 9.00 am Sunday + 10 hours = 7.00 pm Sunday
4 Add the flying time to calculate the time when the plane lands.	Plane arrives at 7.00 pm Sunday + 20 hours = 3.00 pm Monday.

More challenging examples will require you to allow for daylight saving time. When daylight saving time applies, we add one hour to the standard time at that location.

remember

1. Time zones are calculated by comparison with Greenwich Mean Time (GMT).
2. For places to the east we add time, for places to the west we subtract time to calculate the time zone.
3. The time zone is calculated using the longitude.
   - 1 hour = 15° of longitude
   - 1° of longitude = 4 minutes
4. To calculate the time, subtract the time when heading west and add time when heading east.
5. Care must be taken when crossing the International Date Line as you will need to add a day when travelling east or subtract a day when heading west.
6. Time differences need to be used when calculating departure time and arrival time for international travel.
7. When daylight saving time applies, we add one hour to the standard time.

EXERCISE 13E Time zones

SKILLSHEET 13.3 Converting units of time

WORKED Example 12

1 The time zone in New Zealand is GMT +12 while in Turkey it is GMT +2. Calculate the time difference between New Zealand and Turkey.

2 Calculate the time difference between each of the following locations.
- a Tokyo GMT +9 and New York GMT −5
- b Los Angeles GMT −8 and Dakar GMT −1
- c Rio De Janeiro GMT −3 and Perth GMT +8
- d Hawaii GMT −11 and Fiji GMT +11

WORKED Example 13

3 Sydney is GMT +10, while San Francisco is GMT −8. When it is 5.00 pm on Tuesday in Sydney, what is the time in San Francisco?

4 For each of the following calculate:
- a the time in Perth (GMT +8) when it is 10.00 pm in Sydney (GMT +10)
- b the time in Washington (GMT −5) when it is 4.00 am Saturday in Sydney (GMT +10)
- c the time in Auckland (GMT +12) when it is 7.00 am Wednesday in Johannesburg (GMT +2)
- d the time in Sydney (GMT +10) when it is 6.00 am Tuesday in Salt Lake City (GMT −7)
- e the time in Adelaide (GMT +9.5) when it is 8.15 pm Sunday in the Cook Islands (GMT −10).

5 Jane is in Sydney (GMT +10) and wants to telephone her friend in Paris (GMT) at 7.00 pm Friday, Paris time. At what time must she call from Sydney?

6 Carl is holidaying in Hawaii (GMT −11). If he wants to call his parents in Sydney (GMT +10) at 8.00 pm on Wednesday, what time must he call from Hawaii?

7 Neville is in Sydney (GMT +10). He wants to set his video recorder to tape the Superbowl which is being played in Atlanta (GMT −5) and televised live in Sydney. The Superbowl is due to begin at 7.00 pm on Sunday in Atlanta. At what day and time will Neville need to set his video to begin taping?

8 Sydney is GMT +10.
- **a** What is the time in Sydney when it is noon GMT?
- **b** When daylight saving time is applied, describe the time zone in Sydney as compared to GMT.
- **c** During daylight saving time, what will the time be in Sydney when it is:
  - **i** 4.00 am Monday GMT?
  - **ii** 9.00 pm Thursday GMT?

9 Sydney is GMT +10 and Los Angeles is GMT −8. Calculate the time difference between Sydney and Los Angeles when:
- **a** both cities are on standard time
- **b** Sydney has daylight saving time and Los Angeles is on standard time
- **c** Los Angeles has daylight saving time and Sydney is on standard time.

WORKED Example 14

10 Beijing is at approximately (40°N, 120°E). Rome is at approximately (40°N, 15°E). Calculate the difference in standard time between Beijing and Rome.

11 Calculate the time difference in standard time between each of the following cities.
- **a** Mumbai (19°N, 73°E) and Casablanca (23°N, 82°W)
- **b** Tokyo (36°N, 140°E) and Adelaide (23°S, 134°E)
- **c** Miami (26°N, 80°W) and Seattle (47°N, 122°W)

12 **multiple choice**

At a point on the Earth's surface, the coordinates are (45°N, 135°W). The standard time at this point would be:

A GMT −3 **B** GMT +3 **C** GMT −9 **D** GMT +9

13 **multiple choice**

It is 11.00 am Tuesday at a point X with coordinates (32°S, 90°W). At a point, Y, with coordinates (51°N, 120°E), what is the time if daylight saving time applies at Y?

A 9.00 pm Monday **B** 10.00 pm Monday
C 1.00 am Wednesday **D** 2.00 am Wednesday

WORKED Example 15

14 A plane leaves Sydney (32°S, 150°E) at 2.00 pm on Tuesday. If it is an 18-hour flight to Los Angeles (33°N, 120°W), at what time will the plane touch down in Los Angeles?

15 A plane leaves Perth (32°S, 120°E) on an 8-hour flight to Cape Town (33°S, 15°E) at 3.00 pm Wednesday.
- **a** At what time will the plane arrive in Cape Town?
- **b** The return flight leaves Cape Town at 5.00 pm Saturday. At what time will it arrive in Perth?

16 A flight leaves Melbourne (40°S, 150°E) at 5.00 pm Tuesday on an 18-hour flight to Frankfurt (50°N, 15°E). Calculate the time of arrival in Frankfurt if it is:
- **a** daylight saving time in Melbourne
- **b** daylight saving time in Frankfurt.

WORKSHEET 13.2

The keepers of time

Information about the world time zones can be found at the internet site for the Royal Observatory Greenwich, which is recognised worldwide as the keeper of time. Information can be found here about all time zones throughout the world and other facts about time. Their address is www.rog.nmm.ac.uk.

summary

Arc lengths

- The circumference of a circle is found using either $C = \pi d$ or $C = 2\pi r$.
- The length of an arc can be found using the formula:

$$l = \frac{\theta}{360} 2\pi r \quad \text{FS}$$

where θ is the number of degrees in the central angle.

Great circles and small circles

- An axis of the sphere is any diameter. A diameter must pass through the centre of the sphere. The endpoints of the axis are called the *poles*.
- A great circle is any circle of maximum diameter that can be drawn on the surface of a sphere. The circumference of a great circle can be found using the formula $C = 2\pi r$.
- A small circle is any circle on the surface of the sphere that is smaller than a great circle.

Latitude and longitude

- A point on the Earth's surface is located using a pair of coordinates.
- A point is located as being either north or south of the equator using latitude. Latitude is the angle subtended at the centre of the sphere by the small circle on which the point lies.
- A point is located as being either east or west of the Greenwich Meridian by the meridian of longitude. Each meridian of longitude is measured by the angle between the meridian of longitude and the Greenwich Meridian.

Distances on the Earth's surface

- If two points lie on the same great circle, the angular distance between them can be found by finding the difference between their latitudes.
- The distance between two points can then be found in nautical miles using $1° = 60$ M.
- 1 M $\simeq$ 1.852 km.
- The distance between two points can also be found in kilometres using the arc length formula.
- Speed can be measured in knots, where 1 knot = 1 M/h.

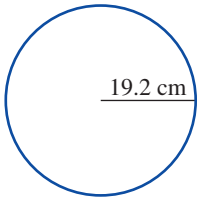
Time zones

- The longitude of a city determines its time zone.
- The standard time at any location can be calculated using 15° = 1 hour or $1° = 4$ minutes.
- Points to the east of the Greenwich Meridian have standard time ahead of GMT. Points to the west of the Greenwich Meridian have standard time behind GMT.

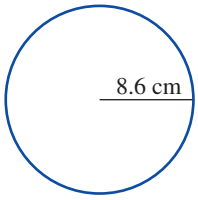
CHAPTER review

1 Calculate the circumference of each of the following circles, correct to 1 decimal place. 13A

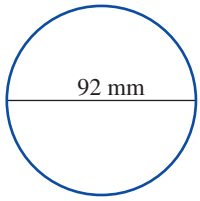
a



b

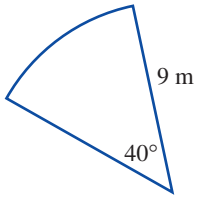


c

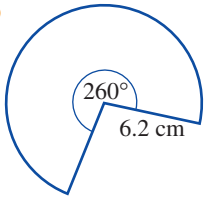


2 Calculate the length of each of the following arcs, correct to 1 decimal place. 13A

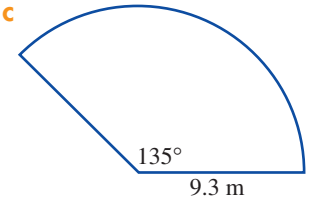
a



b



c



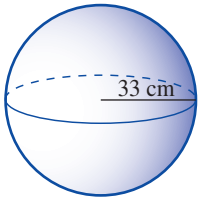
3 A sphere has a radius of 7.5 cm. 13A

a Calculate the circumference of the sphere, correct to 1 decimal place.

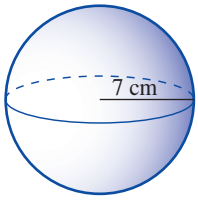
b Calculate the distance between two points on the surface of the sphere that subtend an angle of 60° at the centre. Give your answer correct to 2 decimal places.

4 Calculate the circumference of each of the following spheres, correct to 1 decimal place. 13B

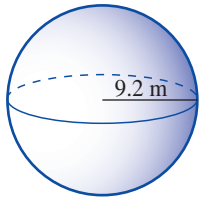
a



b



c

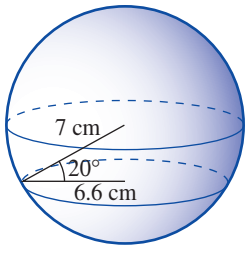


5 Calculate the circumference of a great circle that lies on the surface of a sphere with a radius of 9 km. (Give your answer correct to 1 decimal place.) 13B

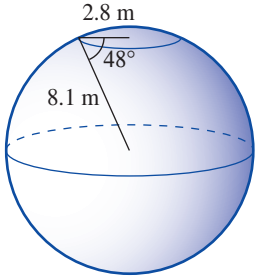
6 A sphere has a diameter of 45 cm. Calculate the distance between the poles on this sphere. Give your answer correct to the nearest centimetre. 13B

7 Calculate the circumference of each of the small circles drawn below, correct to 1 decimal place. 13B

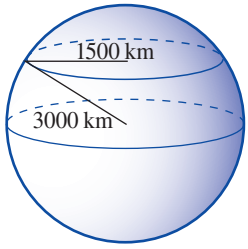
a



b



c



13C

8 Use the world map on page 394 to identify the cities at each of the following locations.
a (14°N, 121°E) **b** (12°S, 76°W) **c** (33°S, 71°W)

13C

9 Use the world map on page 394 to give the approximate coordinates of each of the following cities.
a Madrid **b** Singapore **c** Hobart

13D

10 The points X and Y on the Earth's surface have coordinates (32°N, 120°E) and (26°S, 120°E). Calculate the angular distance between X and Y.

13D

11 A ship gives its coordinates as (56°N, 14°W) and is sailing to a port at (40°N, 14°W).
a Calculate the angular distance through which the ship must sail to reach port.
b Calculate the distance the ship must sail, in nautical miles.
c Use 1 M = 1.852 km to calculate the distance, correct to the nearest kilometre.

13D

12 The angular distance between two points on the same great circle is 120°. Calculate the time that it would take to sail between these points at a speed of 48 knots.

13D

13 Santiago has approximate coordinates (33°S, 70°W), while Santo Domingo has approximate coordinates (18°N, 70°W).
a Calculate the distance between Santiago and Santo Domingo:
i in nautical miles
ii correct to the nearest kilometre.
b Calculate the time taken to fly from Santiago to Santo Domingo at a speed of 480 knots.

13D

14 The coordinates of Perth are approximately (32°S, 115°E), while Hong Kong is approximately at (22°N, 115°E). Taking the radius of the Earth to be 6400 km, use the arc length formula to calculate the distance between Perth and Hong Kong, correct to the nearest 100 km.

13E

15 Calculate the time difference between each of the following cities.
- **a** Sydney (GMT +10) and Istanbul (GMT +2)
- **b** Perth (GMT +8) and New York (GMT −3)
- **c** Ottawa (GMT −5) and Fiji (GMT +12)

13E

16 In Dhahran (GMT +4) the time is 10.00 pm on Wednesday. Calculate the time in Tokyo (GMT +9).

13E

17 Ann is on a skiing holiday in Winnipeg, Canada (GMT −6). She needs to call her parents at 7.30 pm on Tuesday night, Sydney time. At what time should she make the call from Winnipeg?

13E

18 Kingston, Jamaica is at approximately (18°N, 75°W) while Oslo, Norway is at approximately (60°N, 15°E). Calculate the time:
- **a** in Oslo when it is 5.00 am in Kingston
- **b** in Kingston when it is 5.00 pm in Oslo.

13E

19 A plane is flying from Munich (48°N, 15°E) to New York (41°N, 75°W). The flight departs Munich at 6.00 pm and takes 7 hours. Calculate the time of arrival in New York.

Practice examination questions

1 multiple choice

A circle has a diameter of 12 cm. An arc is drawn on the circumference of the circle such that the arc subtends an angle of 45° at the centre of the circle. The length of the arc is closest to:

A 4.7 cm **B** 9.4 cm **C** 14.1 cm **D** 37.7 cm

2 multiple choice

A great circle on the surface of a planet has a circumference of approximately 10 700 km. The diameter of the planet would be closest to:

A 850 km **B** 1700 km **C** 3400 km **D** 6800 km

3 multiple choice

Point X on the Earth's surface has coordinates (69°S, 12°E), while point Y is at (8°S, 12°E). The distance between X and Y is:

A 61 M **B** 77 M **C** 3660 M **D** 4620 M

4 multiple choice

The coordinates of two points, M and N, on the surface of the Earth are (45°N, 45°W) and (30°S, 60°E). Which of the following statements is correct about the time difference between M and N?

A M is 5 hours behind N. **B** M is 5 hours ahead of N.
C M is 7 hours behind N. **D** M is 7 hours ahead of N.

5 The city of St Petersburg is at approximately (60°N, 30°E), while the city of Johannesburg has approximate coordinates of (25°S, 30°E).
- **a** Calculate the angular distance between St Petersburg and Johannesburg.
- **b** Taking the radius of the Earth to be equal to 6400 km, calculate the distance between St Petersburg and Johannesburg, correct to the nearest 100 km.
- **c** In Sydney (30°S, 150°E) daylight saving time applies. Calculate the time difference between St Petersburg and Sydney.

6 The approximate coordinates of Tokyo are (36°N, 140°E), while San Francisco is at approximately (36°N, 120°W).
- **a** Do San Francisco and Tokyo lie on the same great circle or the same small circle? Explain your answer.
- **b** An aeroplane takes 8 hours to fly between Tokyo and San Francisco. If a plane leaves Tokyo at 10.00 pm on Saturday, Tokyo time, what day and time will it arrive in San Francisco?
- **c** If the return flight leaves San Francisco at 8.00 am Tuesday, what day and time will it arrive in Tokyo?